Benchmarking of wind turbine wake models in large offshore wind farms

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Abstract. Three engineering wake models are compared with power production data from the Horns Rev and Lillgrund offshore wind farms. Single and multiple wake cases are investigated to verify the performance of the models in different conditions. The simulations reveal that the three wake models have similar behaviours for both wind farms although the turbine spacing and the turbulence intensity are different. The results prove the robustness of the models to provide accurate power predictions when the simulations are averaged over wind direction sectors of $30°$. However, all models significantly underpredict the power production of a single row of wind turbines using narrow sectors of $3°$ or $5°$. This discrepancy is discussed and justified by the wind direction uncertainty included in the datasets.

1. Introduction

The rising demand for wind power together with social, environmental and economical constraints currently lead to a continuous increase in the size of wind turbines and wind farms. A drawback from installing wind turbines in large arrays is the wake penalty that arises when a wind turbine operates in the lee of a previous one. In large offshore wind farms, the average power loss due to wind turbine wakes is approximately 10 to $20\,\%$ of the annual energy production [1]. As the wind flows through the rotors, turbulent structures are generated and transported downstream. The resulting turbulent velocity field reduces the wind turbine lifespan and increases maintenance costs. Therefore, it is crucial for wind farm developers to estimate accurately the impact of wind turbine wakes because it has become significant for wind farm economics.

The current engineering models such as the Jensen model [2,3] and the eddy viscosity model from Ainslie [4] have proven to be reliable for long-term power predictions in small to medium size wind farms, but tend to underestimate wake losses for large wind farms [5,6]. The simulations from these models are commonly averaged over wind direction sectors of $30°$ to compute the annual energy production (AEP). However, several studies [6–9] showed that model accuracy decreases when wind direction sectors smaller than $10°$ are examined.

With the increase in computer power, some promising approaches have emerged such as computational fluid dynamics (CFD) methods using Reynolds-Averaged Navier-Stokes (RANS) models [9,10] or large eddy simulations (LES) [11–15]. Nevertheless, several studies underlined an intrinsic problem with conventional RANS simulations applied to wind turbine wakes [16–19].

The wide variety of available wake models emphasizes the need for additional validation campaigns to define clear guidelines on how the wind industry shall use the models. The WakeBench project from the International Energy Agency [20] and the European EERA-DTOC project [21] have recently been started to address this issue by comparing wake simulations against various test cases. A better understanding of the model limitations and calibrations will decrease wake loss uncertainty in project development and improve the competitiveness of wind energy.

This study presents a first deliverable to the WakeBench and EERA-DTOC projects by benchmarking the Jensen model, the Larsen model [22] and Fuga [8] with wind farm production data recorded at the Horns Rev and Lillgrund offshore wind farms. These two wind farms enable a verification of the model performances for two different wind turbine spacings and turbulence intensities. An emphasis on the wind direction uncertainty is made in Section 5 to explain the discrepancies for narrow wind direction sectors.

2. Measurements

The Horns Rev wind farm is located in the North Sea 14 km from the west coast of Denmark. It has a total rated capacity of 160 MW and consists of 80 pitch-controlled, variable speed Vestas V80 wind turbines with a rotor diameter of 80 m and a hub height of 70 m. As shown in Figure 1, the wind turbines are positioned in a regular array of 8 by 10 turbines with a spacing along the main directions of 7 rotor diameters (D). The horizontal rows are referred to row A to H while the vertical rows are referred to column 1 to 10. The dataset was quality controlled and processed according to Hansen et al. [23].

The Lillgrund offshore wind farm is located 10 km from the southwestern coast of Sweden between Copenhagen and Malmö. It has a total rated capacity of 110 MW and consists of 48 pitch-controlled, variable speed Siemens SWT-2.3-93 wind turbines with a 92.6 m rotor diameter and a 65 m hub height. The layout of the wind farm is presented in Figure 2. Rows A to H are parallel to the direction 221.8° with a turbine spacing of 4.3 D. Rows 1 to 8 have a turbine spacing of 3.3 D. The data were extracted from the Lillgrund power assessment report from Dahlberg [24].
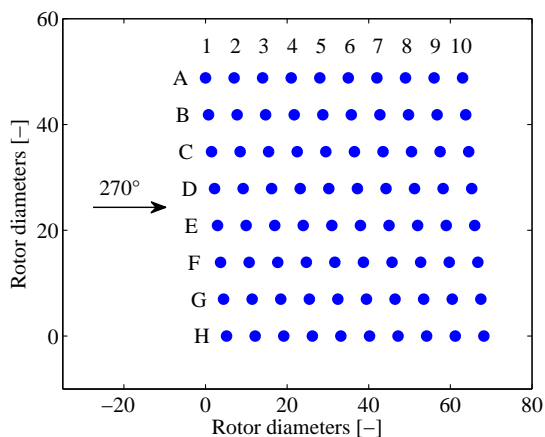

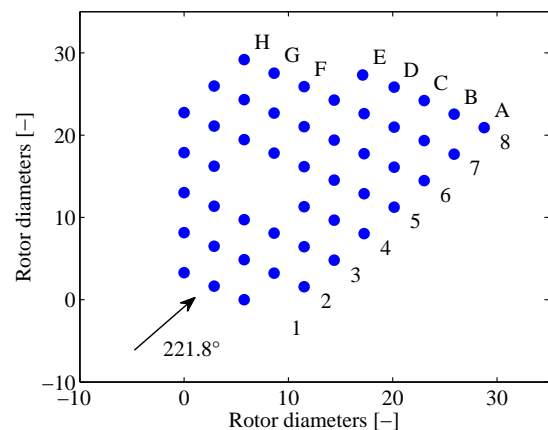**Figure 1.** Layout of the Horns Rev offshore wind farm.

Figure 2. Layout of the Lillgrund offshore wind farm.

3. Wake Models

The Jensen model, the Larsen model and Fuga are the three engineering wake models investigated in this benchmarking study. The three models assume neutral atmospheric stability.

3.1. Jensen Model

The Jensen model here used is a Matlab version of the Park wake model [3] implemented in WAsP [25]. The only difference is that the effect of the ground is not taken into account in the Matlab script. From the law of momentum conservation, an expression for the wake velocity as a function of distance downstream is derived. The initial velocity deficit is calculated from the turbine's thrust coefficient and the rate of wake expansion is determined through an empirical constant (k). In offshore conditions, k is commonly set to 0.05 and 0.04 for small and large wind farms respectively. In this study, the values 0.05 and 0.04 are used for the single wake and multiple wake cases respectively. The total velocity deficit for a given location is calculated as the square root of the sum of squares of the velocity deficits induced by all upstream turbines.

3.2. Larsen Model

The Larsen model corresponds to the most recent update of the model from Larsen [22]. It is implemented in a common platform with the Jensen model in Matlab to facilitate the benchmarking study. The model has a closed-form expression of the wake radius and wake velocity based on the RANS equations and experimental measurements at $9.6\,D$ in the wake of a single turbine. The velocity recovery and wake expansion are controlled by the thrust coefficient and the ambient turbulence intensity. The total velocity deficit for a given location is calculated as the linear sum of the velocity deficits induced by all upstream turbines.

For the wind direction cases investigated in this paper, a turbulence intensity of 7 % and 5.6 % is applied for Horns Rev and Lillgrund respectively. These values are consistent with measurements documented by Hansen et al. [23] and Bergström [26].

3.3. Fuga

Fuga is a linear flow solver based on the steady-state RANS equations. It is designed for flat and homogeneous terrain so its main purpose is wake modelling of offshore wind farms. The flow is assumed incompressible and lid-driven at a chosen height above the ground. The turbines are modelled using an actuator disc technique. The complete equations and their validations against experimental data were recently described by Ott et al. [8].

Fuga is currently implemented as a stand-alone graphical user interface that requires a wind farm layout and wind turbine parameters in WAsP format. A batch version is also available where the user specifies the simulations to perform using command lines.

In this study, Fuga version 2.0.0.28 is used with a roughness length of 0.0001 m and a boundary layer height of 500 m.

4. Results

Figure 3 shows the power of wind turbine G2 normalized to G1 as a function of wind directions relative to 270° at Horns Rev. Fuga and the Jensen model slightly underpredict the power in the interval $[-7°,+7°]$ and overpredict the production for the remaining directions. The Larsen model slightly overpredicts the production for all directions and consistently remains within the error bars ($\pm$ 0.5 standard deviation). The shape of the power deficit is best captured by the Larsen model.

Figure 4 shows the power of wind turbine C2 normalized to C1 as a function of wind directions relative to 221.8° at Lillgrund. It can be noticed that the power deficit is higher at Lillgrund than at Horns Rev due to the shorter turbine spacing. In this case, the Larsen model significantly
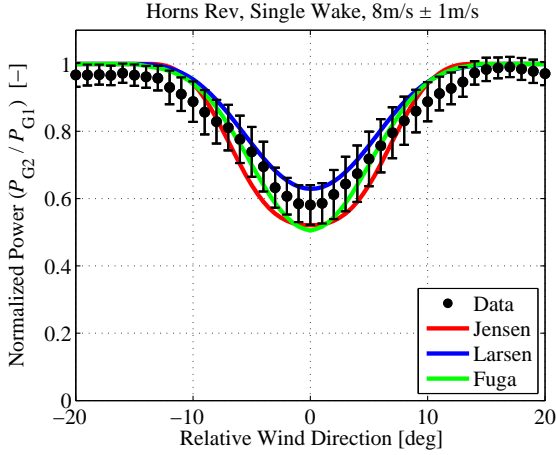
Figure 3. Normalized power of G2 as a function of wind directions relative to 270° at Horns Rev.
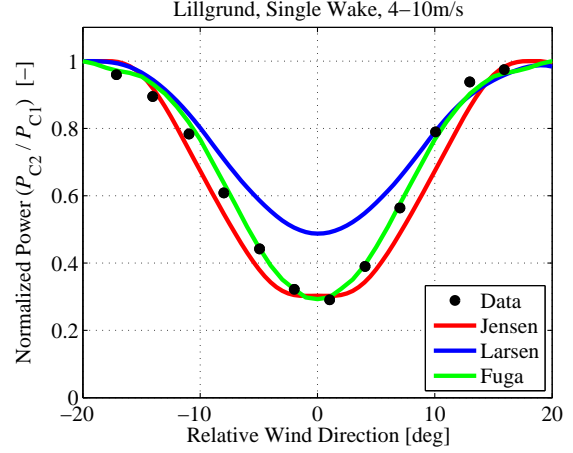
Figure 4. Normalized power of C2 as a function of wind directions relative to 221.8° at Lillgrund.

overpredicts the production. The Jensen model and Fuga obtain excellent agreement both in terms of the deficit amplitude and the shape. In fact, Fuga matches almost perfectly all data points. All three models predict accurately the width of the power deficit with a recovery to $P_{C2}/P_{C1} = 1$ at $\pm 15°$.

It is worth mentioning that a moving average of 5° and 3° was applied on the simulations in Figures 3 and 4 respectively to be consistent with the data processing [23, 24].

Figures 5 and 6 show the normalized power production in row E at Horns Rev for two different averaging sectors. The trend for the three models is the same. They underpredict the production of the narrow sector ($\pm 2.5°$), while obtaining good to excellent accuracy for $\pm 15°$.

Figures 7 and 8 present the normalized power production in row C at Lillgrund for two different averaging sectors. Similarly to Horns Rev simulations, the three models underpredict the production for the narrow sector ($\pm 1.5°$) and obtain good to excellent agreement for $\pm 15°$. Hence, there is a clear correlation between the accuracy of the predictions and the span of the averaging sector.

5. Discussion

Figures 3 and 4 reveal that the Larsen model differs more from the data for the Lillgrund single wake test case. This discrepancy can be explained by the short turbine spacing at Lillgrund. More precisely, the turbine spacing between turbine C1 and C2 is $4.3\,D$, which is much shorter than the $9.6\,D$ used to calibrate the model. Alternatively, the Horns Rev test case is more suitable for the Larsen model because the turbine spacing is $7\,D$. Yet, figures 5 to 8 demonstrate that the overpredictions of the Larsen model for the single wake situations vanish in multiple wake situations.

Figure 6 shows that the Jensen model and Fuga simulate correctly the production of turbine E2 for the averaging sector $\pm 15°$. This result proves that the underpredictions made in the interval $[-7°, +7°]$ in Figure 3 are balanced by the overpredictions of the remaining directions. Therefore, it is an example of two modelling errors with opposite sign that converge towards the right solution. This finding confirms a recent study performed by Beaucage et al. [6] where they suggested that the "errors generated under flow conditions directly parallel to the rows are balanced by errors of the opposite sign for flow oblique to the rows".

Another example of balancing errors occurs for the Jensen model in Figures 6 and 8. For both
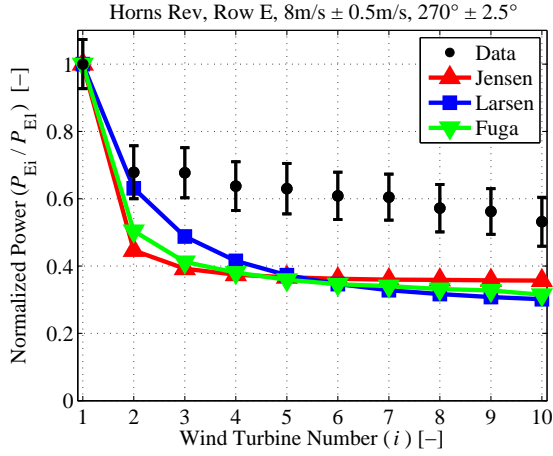
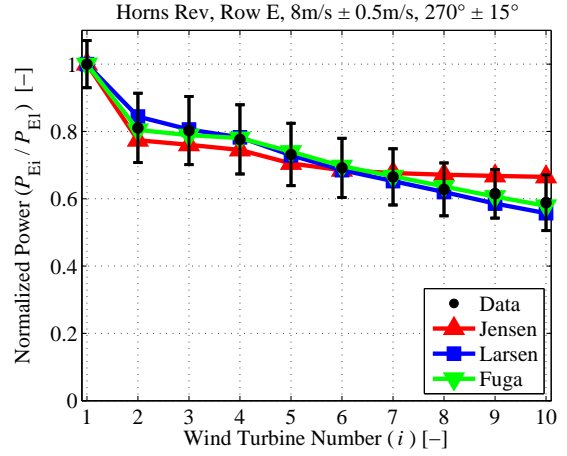**Figure 5.** Normalized power in row E at Horns Rev for the wind direction sector $270° \pm 2.5°$.

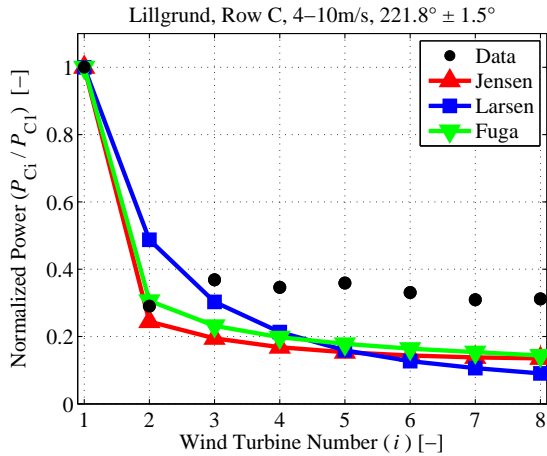**Figure 6.** Normalized power in row E at Horns Rev for the wind direction sector $270° \pm 15°$.

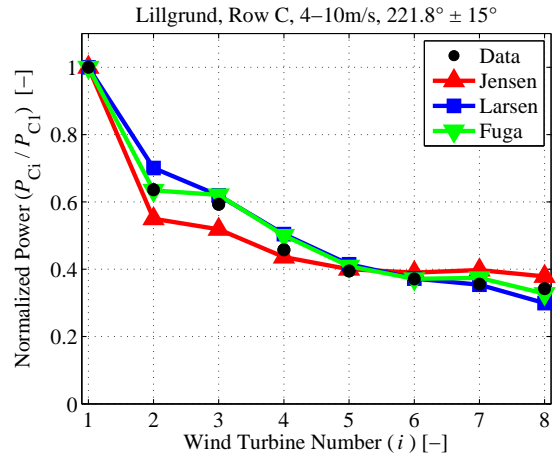**Figure 7.** Normalized power in row C at Lillgrund for the wind direction sector $221.8° \pm 1.5°$.

Figure 8. Normalized power in row C at Lillgrund for the wind direction sector $221.8° \pm 15°$.

wind farms, the model underpredicts the production of the first turbines while overpredicting the last ones. Nevertheless, Table 1 shows that the predicted efficiency of the complete rows with the Jensen model is accurate. Fuga and the Larsen model reproduce the reduction of power in the rows for wide sectors. If the rows were longer, these two models would potentially still make accurate predictions, whereas the Jensen model would probably overpredict significantly the last turbines. This aspect is of primary importance in a context where the size of wind farms is increasing every year. The overprediction of the Jensen model can be mitigated by reducing the value of k for deeper turbines in the array [27].

Figures 5 and 7 demonstrate that none of the models is able to predict the power of a row corresponding to a narrow wind direction sector. Similar results were reported in previous studies [6–9]. Gaumond [7] showed that the discrepancy between the numerical simulations and the power production data is not caused by a fundamental inaccuracy of the models, but rather by a large wind direction uncertainty in the dataset.

Table 1. Measured and predicted efficiency of row E and C at Horns Rev and Lillgrund, respectively, corresponding to the $\pm 15°$ sector.

	Data	Jensen	Larsen	Fuga
Horns Rev, Row E	73.1 %	+0.4 %	−0.5 %	−0.1 %
Lillgrund, Row C	52.0 %	−1.1 %	+1.3 %	+1.0 %

5.1. Wind Direction Uncertainty

Three main sources of wind direction uncertainty can be identified in the datasets, namely the yaw misalignment of the reference wind turbine, spatial variability of the wind direction within the wind farm and the wind direction averaging period.

Hansen et al. [23] and Dahlberg [24] used the yaw position sensor of an upstream wind turbine to determine the undisturbed wind direction. Hansen et al. [23] mentioned that using a turbine yaw position as reference "results in an uncertainty of more than 7° because the yaw misalignment also needs to be included". Indeed, the control strategy responsible for yawing the turbine does not respond instantaneously to natural wind variations. In addition, the offset of the yaw position sensor determined during data post-processing might vary over a few years of wind farm operation. Hence, the corresponding reference wind direction used to bin the data might not be accurately representative of the whole dataset.

Horns Rev and Lillgrund are large wind farms where the wind direction is not strictly constant and homogeneous due to natural variations of the wind over few kilometers. As the distance between the reference turbine and the others increases, the wind direction correlation decreases. In consequence, spatial variability increases the wind direction uncertainty.

The datasets of Horns Rev and Lillgrund represent 10-minute and 1-minute averages respectively. Due to the stochastic behaviour of the wind, natural fluctuations of wind direction occur during these averaging periods. In theory, the prescribed turbulence intensity and rate of wake expansion should account at least partly for the variations of the wind direction within one averaging period. However, the drift of the wind direction between two averaging periods is not modeled in the simulations because it corresponds to large-scale weather phenomena. Therefore, a fraction of the random behaviour of the wind direction within one averaging period is not modeled and represents a wind direction uncertainty included in the datasets.

The averaging time of 1 minute used for the Lillgrund's dataset has the disadvantage of increasing the uncertainty from spatial coherence. Indeed, the wind needs approximately 1.5 minute at 9 m/s to reach the third turbine in row C. Therefore, the turbines C1 and C3 to C8 can be operating under different wind conditions for the same 1-minute period, which results in a higher wind direction uncertainty for these turbines. The fact that turbine C2 is more correlated with C1 might explain why its power is captured more accurately than the others in Figure 7.

Due to this wind direction uncertainty, narrow wind direction bins of 5° (Figure 5) and 3° (Figure 7) most likely include situations where the turbines operate in conditions outside the span of the sector. In this case, it means that the turbines operate more in wake free or partial wake situations (i.e. higher power output) than what is modelled by the numerical simulations. In turn, when the sector width is increased the uncertainty becomes less significant and less cases are filtered in the wrong bins. Thus, the agreement in Figures 6 and 8 is improved because the simulations for wide sectors are more representative of the datasets. It can be understood that the correlation between the accuracy of the model and the width of the wind direction sector is caused by the wind direction uncertainty.

A method to address the wind direction uncertainty and thereby improve the agreement of the numerical simulations for narrow wind direction sectors was proposed by Gaumond [7, 28]. The idea is to replace a single simulation performed with a fixed wind direction by a weighted average of several simulations performed for a wide spectrum of wind directions.

6. Conclusion
The Jensen model, the Larsen model and Fuga perform similarly at Lillgrund and Horns Rev although the turbine spacing and the turbulence intensity are different. The results prove the robustness of the three models to provide accurate predictions within a $1.5\,\%$ error margin for wide sectors of $30°$. However, all models underpredict the power production of a row of wind turbine using narrow sectors of $3°$ or $5°$. This discrepancy is caused by the wind direction uncertainty which is not modelled in the simulations.

Fuga and the Larsen model are expected to make accurate predictions for larger wind farms than Horns Rev and Lillgrund because they capture accurately the reduction of power along a row of wind turbines. The Jensen model will overpredict significantly the production of the last turbines with the current recommended parameter for large offshore wind farm ($k = 0.04$).

Future work shall investigate if a reliable and consistent method can quantify the wind direction uncertainty in large wind farms to enable a better and more fair comparison between power production data and numerical simulations.

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